

Isovector response and energy-weighted sums in hot nuclei

V. M. Kolomietz, S. V. Lukyanov, O. I. Davidovskaya
Institute for Nuclear Research, 03680 Kyiv, Ukraine

Abstract

We investigate the collective response function and the energy-weighted sums (EWS) m_k for isovector mode in hot nuclei. The approach is based on the collisional kinetic theory and takes into consideration the temperature and the relaxation effects. We have evaluated the temperature dependence of the adiabatic, $E_1 = \sqrt{m_1/m_{-1}}$, and scaling, $E_3 = \sqrt{m_3/m_1}$, energy centroids of the isovector giant dipole resonances (IVGDR). The centroid energy E_3 is significantly influenced by the Fermi surface distortion effects and, in contrast to the isoscalar mode, shows much weaker variation with temperature. Taking into account a connection between the isovector sound mode and the corresponding surface vibrations we have established the A -dependence of the IVGDR centroid energy which is in a good agreement with experimental data. We have shown that the enhancement factor for the "model independent" sum m_1 is only slightly sensitive to the temperature change.

Keywords: Fermi system, kinetic theory, response function, energy weighted sum, isovector giant dipole resonance, relaxation, temperature

PACS: 21.60Jz, 24.30.Cz, 26.60.Ev, 24.10.Nz

1 Introduction

Many features of nuclei are sensitive to nuclear heating. The nuclear heating influences strongly the particle distribution near the Fermi surface and reduces the Fermi-surface distortion effects on the nuclear collective dynamics [1]. Moreover, the heating of the nucleus provides the transition from the rare- to frequent interparticle collision regime. One can expect that the zero-sound excitation modes which exist in cold nuclei will be transformed to the first-sound ones in hot nuclei. Knowledge of the nuclear collective dynamics in hot nuclei allows one to understand a number of interesting phenomena, e.g., the temperature dependence of the basic characteristics of isovector giant dipole resonance (IVGDR). The existence of the IVGDR in the heated nuclei built on the excited states was established a long time ago [2]-[7]. A systematic experimental and theoretical study of the IVGDR in hot nuclei provides a significant information on the isovector collective motion at non-zero temperatures [8, 9].

A good first orientation in a description of the IVGDR in hot nuclei is given by a study of the isovector response and the relevant energy weighted sums within the quantum RPA-like approaches [10]-[14] or the semiclassical kinetic theory [15]-[18]. In the present work, following the ideology of the kinetic theory, we consider both temperature and relaxation effects on the IVGDR characteristics. Our semiclassical kinetic approach ignores both the shell and single particle spin effects. Nevertheless, it seems to be quite instructive for an investigation of the averaged properties of the many-body systems. In many cases, it allows us to obtain analytical results and represent them in a transparent way. There are also some conceptual advantages in the use of the kinetic theory. Kinetic approach involves the temperature directly into the equations of motion for the distribution function, e.g., the temperature is considered here as a dynamic variable. In contrast, in quantum approaches the temperature appears after ensemble smearing of the observable quantities and can not be attributed to the equation of motion for the wave function. Moreover the kinetic approach can be easily generalized to consider the relaxation (damping) processes by introduce of the collision integral [19, 20]. Note that a similar extension of the RPA due to the involving of the coupling with $2p - 2h$ states provides only the fine structure of the giant multipole resonance (GMR) [21, 22] and an additional smearing procedure for the strength function has to be used to derive the corresponding collisional width of the GMR. Note also that the non-collisional fragmentation width of the GMR (spreading of the GMR over non-collective $1p - 1h$ excitations in the RPA) does not drive the system toward a thermal equilibrium but rather indicates a redistribution of the particle-hole excitations in the vicinity of the collective state [13]. In the kinetic approach, this mechanism is presented due to the Landau damping.

In what follows we use the kinetic approach to study both the temperature and mass number dependencies of the averaged characteristics of the IVGDR, such as centroid energies, width, isospin symmetry energy, mass coefficient, energy weighted sum (EWS) and EWS enhancement factor. The corresponding analysis within RPA and beyond RPA requires a large amount of numerical calculations, see e.g. [23], which not necessary provide a clear understanding of above mentioned macroscopic features of the IVGDR. Both the quantum RPA and the kinetic approaches use the effective nucleon-nucleon interaction. We apply the effective Landau interaction in nuclear interior and the macroscopic boundary condition in surface region. The macroscopic boundary condition includes the phenomenological surface tension coefficient and thereby substitutes for an effective interaction within the surface layer. Note that the effective interaction is usually not quite well-defined near nuclear surface because of strong particle density inhomogeneity in this case and the involving of the relevant boundary conditions can be used to improve the description. Our goal is also to study the conditions for zero- to first- sound transition in presence of the velocity-dependent forces and the effect of the thermal Landau damping [15] on the low-energy tail of the strength function under different temperature and relaxation conditions. Similar problems were considered in Refs. [16]-[18] by neglecting the velocity-dependent part of Landau forces and using the simplest boundary condition of Steinwedel-Jensen (SJ) model for the wave number $k = \pi/2R_0$, where R_0 is the nuclear radius. However, it is well known [24] that such kind of boundary condition do not allow the correct description of the A -dependence of the IVGDR energy for light nuclei. A prove of the modified boundary condition for the isovector

eigenmode for a finite Fermi-liquid drop plies an important role in our consideration.

There are different theoretical approaches to describe the temperature behavior of the IVGDR width. Below, we will restrict ourselves by the collisional damping and the thermal Landau spreading. The alternative approach is the thermal fluctuation model (TFM) in the adiabatic coupling scheme [25] which explains the temperature increase of the IVGDR width as an effect of the adiabatic coupling of the IVGDR to thermal shape fluctuations. One can expect that the combine of both approaches will be able to provide a satisfactory description of both the temperature and the mass number dependencies of the IVGDR width [26]. This is an object of our forthcoming investigation.

Finally, we would like to note that we use the IVGDR as an instrument to study the isovector motion in a spherical nuclear Fermi-liquid drop. Our claim is to describe the general features of collective excitations, such as the A -dependence of the IVGDR energy and the isovector sound mode in nuclear Fermi liquid ignoring many quantum effects. In particular, our approach provides the possibility to compare the predictions of the standard liquid drop model with the Fermi liquid drop one where the dynamic distortions of Fermi surface is taking into account.

In Section 2 we apply the Landau's kinetic theory to the evaluation of the response function in a two-component nuclear Fermi liquid. Both the temperature and the relaxation phenomena are taken into account. In Section 3 we derive the energy weighted sums as the moments of the strength function. Numerical illustrations, summary and conclusions are given in Sections 4 and 5.

2 Response function within the kinetic theory

To derive the energy-weighted sums for the isovector excitations, we will consider the density-density response of two-component nuclear matter to the following external field

$$U_{\text{ext}}(t) = \lambda_0 e^{-i\omega t} \hat{q} + \lambda_0^* e^{i\omega t} \hat{q}^*, \quad (1)$$

where λ_0 is the small amplitude, \hat{q} is the one-body operator

$$\hat{q} = \sum_{j=1}^A \hat{q}(\vec{r}_j, \xi_j) = \sum_{j=1}^A \xi_j e^{-i\vec{q} \cdot \vec{r}_j}$$

and ξ_j is the isotopic index ($\xi = p$ (or $+1$) for proton and $\xi = n$ (or -1) for neutron). The response density-density function $\chi(\omega)$ is given by [27]

$$\chi(\omega) = \frac{\langle e^{-i\vec{q} \cdot \vec{r}} \rangle}{\lambda_0 e^{-i\omega t}} = \frac{1}{\lambda_0 e^{-i\omega t}} \int d\vec{r} e^{-i\vec{q} \cdot \vec{r}} \delta\rho_-(\vec{r}, t), \quad (2)$$

where the isovector particle density variation $\delta\rho_-(\vec{r}, t) \equiv \delta\rho_- = \delta\rho_n - \delta\rho_p$ is due to the external field $U_{\text{ext}}(t)$ of Eq. (1).

Below we will apply the kinetic theory in (\vec{r}, \vec{p}) phase space to the evaluation of the response function of Eq. (2). The particle density variation $\delta\rho_-$ is then given by

$$\delta\rho_-(\vec{r}, t) = \int \frac{g d\vec{p}}{(2\pi\hbar)^3} \delta f_-(\vec{r}, \vec{p}; t). \quad (3)$$

Here, $g = 2$ is the spin degeneracy factor and $\delta f_-(\vec{r}, \vec{p}; t) \equiv \delta f_- = \delta f_n - \delta f_p$ is the isovector deviation of the distribution function $f(\vec{r}, \vec{p}; t)$ from the equilibrium one at certain temperature T

$$f_{\text{eq}}(\epsilon_p) = \left[1 + \exp \frac{\epsilon_p - \mu}{T} \right]^{-1}, \quad (4)$$

where μ and $\epsilon_p = p^2/2m^*$ are respectively the quasiparticle chemical potential and energy, m^* is the effective nucleonic mass.

A small isovector variation of the distribution function δf_- in Eq. (2) can be evaluated using the linearized kinetic equation. In the nuclear volume, where inhomogeneity of the particle density is small, the quasiparticle concept of the Landau-Fermi-liquid theory [28] can be justified. To evaluate δf_- we will apply the linearized Landau-Vlasov equation, completed by a source term $\delta St(f)$ for relaxation processes in the following form [19, 20]

$$\frac{\partial}{\partial t} \delta f_- + \vec{v} \cdot \nabla_{\vec{r}} \delta f_- - \nabla_{\vec{r}} (\delta U_{\text{self}} + U_{\text{ext}}) \cdot \nabla_{\vec{p}} f_{\text{eq}} = \delta St[f], \quad (5)$$

where $\vec{v} = \vec{p}/m^*$ is the quasiparticle velocity. We point out that the left hand side of kinetic equation (5) can be derived by the Wigner transformation from the linearized time dependent Hartree-Fock equation in presence of the external field U_{ext} [29]. The variation of the selfconsistent mean field δU_{self} in Eq. (5) is then given by the Wigner transformation to the corresponding mean field of the RPA. The selfconsistent mean field δU_{self} is related to the Skyrme or Landau effective interaction [30]. Within the Landau-Fermi-liquid theory the quantity δU_{self} can be derived in terms of the Landau's interaction amplitude $v_{\text{int}}(\vec{p}, \vec{p}')$ as [19, 20, 31]

$$\delta U_{\text{self}} = \int \frac{g d\vec{p}'}{N_T (2\pi\hbar)^3} v_{\text{int}}(\vec{p}, \vec{p}') \delta f_-(\vec{r}, \vec{p}'; t). \quad (6)$$

The interaction amplitude $v_{\text{int}}(\vec{p}, \vec{p}')$ is parameterized in terms of the Landau constants F'_l as

$$v_{\text{int}}(\vec{p}, \vec{p}') = \sum_{l=0}^{\infty} F'_l P_l(\hat{p} \cdot \hat{p}'), \quad \hat{p} = \vec{p}/p \quad (7)$$

and the thermally averaged density of states N_T in Eq. (6) is introduced to provide the dimensionless constants F'_l in Eq. (7). Namely,

$$N_T = - \int \frac{g d\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p}, \quad (8)$$

with $N_0 = g p_F m^*/2\pi^2 \hbar^3$, where p_F is the Fermi momentum.

The right-hand side $\delta\text{St}[f]$ of Eq. (5) is the Uehling-Uhlenbeck type collision integral linearized in δf_- . The collision integral $\delta\text{St}[f]$ depends on the transition probability of the two-nucleon scattering with initial momenta (\vec{p}_1, \vec{p}_2) and final momenta (\vec{p}'_1, \vec{p}'_2) . At low temperatures $T \ll \epsilon_F$, where ϵ_F is the Fermi energy, the momenta (\vec{p}_1, \vec{p}_2) and (\vec{p}'_1, \vec{p}'_2) are localized near the Fermi surface and the relaxation time approximation can be used, see Refs. [19, 20, 28],

$$\delta\text{St}[f] = - \sum_{lm, l \geq 1}^{\infty} \frac{1}{\tau_l} \delta f_{-lm}, \quad (9)$$

where τ_l is the collisional relaxation time and δf_{-lm} is a component of the l, m multipolarity in \vec{p} -space of the isovector variation δf_- . Below we will restrict ourselves to the azimuthally symmetric case (longitudinal perturbation) where δf_- depends only on the angle θ_{pq} between \vec{p} and \vec{q} , i.e. δf_{-lm} is m -independent, see comment after Eq. (14). The partial relaxation time τ_l in Eq. (9), which corresponds to the Fermi-surface distortion of multipolarity l , is derived in this case as [31]

$$\frac{1}{\tau_l} = - \frac{\int_0^\infty d\epsilon_p \int d\Omega_p Y_{l0}^*(\hat{p}) \delta\text{St}[f]}{\int_0^\infty d\epsilon_p \int d\Omega_p Y_{l0}^*(\hat{p}) \delta f_-}. \quad (10)$$

Note that there is no term with $l = 0$ in the sum (9) because of the conservation relation for the particle number in collision processes. In contrast to the case of isoscalar mode, the inclusion of the $l = 1$ term in the collision integral of Eq. (9) is due to the nonconservation of the isovector current, i.e. due to the pn collisions for the counterstreaming neutron and proton flows. The numerical analysis shows [16, 17] that the isovector relaxation time τ_l depends only slightly on the multipolarity $l \geq 2$ and we will use below the following form for the collision integral

$$\delta\text{St}[f] = -\frac{1}{\tau_1} \delta f_-|_{l=1} - \frac{1}{\tau_2} \delta f_-|_{l \geq 2}. \quad (11)$$

Here the notations $l = 1$ and $l \geq 2$ mean that the perturbation of $\delta f_-|_{l=1}$ and $\delta f_-|_{l \geq 2}$ in the collision integral includes only Fermi surface distortions with a multipolarity $l = 1$ and $l \geq 2$, respectively.

The collisional relaxation time τ_l in Eq. (10) is temperature and frequency dependent. The temperature dependence of τ_l arises from the smeared out behavior of the equilibrium distribution function f_{eq} , see Eq. (4), near the Fermi momentum [19, 28]. The frequency dependence of τ_l is caused by the memory (non-Markovian) effect in the collision integral. It can be shown, see Ch. 8 of Ref. [28], that the presence of fast collective mode changes the energy conservation factor in the collision integral $\delta\text{St}[f]$ and provides the frequency dependence of the collisional relaxation time τ_l . Following Landau's prescription [28], we will assume, see also Refs. [16, 17, 32, 33],

$$\tau_l = \frac{\hbar \alpha_l}{T^2 + (\hbar \omega / 2\pi)^2}. \quad (12)$$

The parameter α_l in Eq. (12) depends on the NN -scattering cross sections. In the case of isotropic energy independent cross sections the result for α_1 and α_2 reads [33, 34]

$$\alpha_1 = 3 \epsilon_F^2 / 4 \pi^2 \hbar \rho_{\text{eq}} v_F \sigma_-, \quad \alpha_2 = 5 \epsilon_F^2 / 4 \pi^2 \hbar \rho_{\text{eq}} v_F \sigma_{\text{av}}, \quad (13)$$

where ϵ_F is the Fermi energy, $v_F = p_F / m^*$ and ρ_{eq} is the bulk density in the nuclear interior. The NN -scattering cross sections σ_{av} and σ_- in Eq. (13) are given by $\sigma_{\text{av}} = (\sigma_{\text{pp}} + \sigma_{\text{nn}} + 2 \sigma_{\text{pn}}) / 4$ and $\sigma_- = \sigma_{\text{pn}} / 2$, where σ_{pp} , σ_{nn} and σ_{pn} are the cross sections for nucleon pairs with relative kinetic energy close to the Fermi energy. The value of α_l is significantly different for both vacuum and in-medium reduced cross sections. Using the vacuum NN cross sections [35] $\sigma_{\text{pp}} = \sigma_{\text{nn}} = 2.5 \div 2.7 \text{ fm}^2$ and $\sigma_{\text{pn}} = \sigma_{\text{np}} = 4.8 \div 5.0 \text{ fm}^2$, one obtains the following estimate of $\alpha_{2,\text{vac}} = 2.2 \div 2.3 \text{ MeV}$. The vacuum cross section is more appropriate in the surface layer of the nucleus. Due to the Pauli blocking effect one can expect that the NN cross sections in nuclear matter should be lower than the one in free space. Unfortunately, there is a strong uncertainty in the derivation of the in-medium reduced NN cross sections [35]. We will use the the following in-medium estimate of $\alpha_{2,\text{bulk}} = 5.4 \text{ MeV}$, see Refs. [16, 33].

In general the partial relaxation time τ_2 in Eq. (11) is larger than τ_1 . It is convenient to introduce the relation $\alpha_1 = \alpha_2 / (1 - \eta)$, where η is the dimensionless parameter. In the case of $\eta \rightarrow 1$ and $\alpha_1 \rightarrow \infty$, the relative motion of the proton-neutron fluids is not damped. The character of damping of the isovector mode depends on the sign of parameter η . The zero-to-first sound transition is only possible for $\eta > 0$ [17]. For $\eta < 0$, the relaxation due to τ_1 leads to the faster equilibration of the out phase proton-neutron motion than the transition to the first sound.

At low temperatures only \vec{p} near the Fermi surface enter δf_- and the solution of Eq. (5) can be found in the form

$$\delta f_-(\vec{r}, \vec{p}; t) = - \frac{\partial f_{\text{eq}}}{\partial \epsilon_p} \nu_{\omega, \vec{q}}(\vec{p}) e^{i(\vec{q} \cdot \vec{r} - \omega t)}, \quad (14)$$

where $\partial f_{\text{eq}} / \partial \epsilon_p$ is a sharply peaked function at $p = p_F$ and $\nu_{\omega, \vec{q}}(\vec{p})$ depends only on the direction of \vec{p} . Moreover we consider the azimuthally symmetric case where $\nu_{\omega, \vec{q}}(\vec{p})$ depends only on the angle θ_{pq} between \vec{p} and \vec{q} , and expand $\nu_{\omega, \vec{q}}(\vec{p})$ in Legendre polynomials as

$$\nu_{\omega, \vec{q}}(\vec{p}) = \sum_{l=0}^{\infty} P_l(\cos \theta_{pq}) \nu_l(p). \quad (15)$$

Using Eqs. (5), (6), (14) and (15), we obtain

$$\begin{aligned} & \left[\left(\omega + \frac{i}{\tau_2} \right) - \vec{q} \cdot \vec{v} \right] \nu_{\omega, \vec{q}}(\vec{p}) + \vec{q} \cdot \vec{v} \frac{1}{N_T} \int \frac{g d\vec{p}'}{(2\pi\hbar)^3} v_{\text{int}}(\vec{p}, \vec{p}') \frac{\partial f_{\text{eq}}}{\partial \epsilon_p} \nu_{\omega, \vec{q}}(\vec{p}') \\ & + \lambda_0 \vec{q} \cdot \vec{v} = \frac{i}{\tau_2} \nu_0(p) P_0(\cos \theta_{pq}) + \eta \frac{i}{\tau_2} \nu_1(p) P_1(\cos \theta_{pq}). \end{aligned} \quad (16)$$

Substituting expressions (7) and (15) into Eq. (16) and performing integration in Eq. (16) over \vec{p} , one can come to the following set of equations for the amplitudes $\nu_l(p)$:

$$\begin{aligned} & \nu_l(p) + (2l+1) \sum_{l'=0}^{\infty} \frac{F'_{l'}}{2l'+1} \tilde{\nu}_{l'} Q_{ll'}(z) - \lambda_0(2l+1) Q_{l0}(z) \\ &= i(2l+1) \gamma \nu_0(p) \frac{1}{z} [\delta_{l0} - Q_{l0}(z)] - i(2l+1) \eta \gamma \nu_1(p) Q_{l0}(z). \end{aligned} \quad (17)$$

Here, $\tilde{\nu}_l$ is the averaged amplitude

$$\tilde{\nu}_l = -\frac{1}{N_T} \int \frac{g d\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \nu_l(p), \quad (18)$$

and

$$Q_{ll'}(z) = -\frac{1}{2} \int_{-1}^1 dx \frac{P_l(x) x P_{l'}(x)}{z-x}, \quad x = \cos \theta_{pq}, \quad \gamma = \frac{1}{\tau_2 q v}, \quad z = s + i\gamma, \quad s = \frac{\omega}{qv}.$$

For simplicity we will assume

$$F'_{l=0} \neq 0, \quad F'_{l=1} \neq 0, \quad F'_{l \geq 2} = 0. \quad (19)$$

Under the condition (19), the basic equations (17) can be solved with respect to the amplitude $\tilde{\nu}_0$. After rather simple calculation one can obtain (see Appendix A)

$$\tilde{\nu}_0 = \frac{\tilde{\chi}_{\text{in}}(\omega, q)}{1 + F'_0 \tilde{\chi}_{\text{in}}(\omega, q)} \lambda_0, \quad (20)$$

where the internal response function $\tilde{\chi}_{\text{in}}(\omega, q)$ is given by Eq. (A 6).

The amplitude $\tilde{\nu}_0$ is related to the density-density response function of Eq. (2). Substituting Eq. (14) into Eq. (3) and using Eq. (15), one obtains

$$\delta \rho_{-}(\vec{r}, t) = - \int \frac{g d\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \nu_0(p) e^{i(\vec{q} \cdot \vec{r} - \omega t)}. \quad (21)$$

Using definition (2) and Eqs. (18) and (21), we obtain the density-density response function $\chi(\omega, q)$ for a given momentum transfer q in the following form

$$\chi(\omega, q) = \frac{2 N_T \tilde{\chi}_{\text{in}}(\omega, q)}{1 + F'_0 \tilde{\chi}_{\text{in}}(\omega, q)}. \quad (22)$$

Equation (22) (together with (A 6)) gives a generalization of analogous result of Refs. [16, 17] to the case of the velocity dependent (nonlocal) interaction $F'_1 \neq 0$. The poles of the response function $\chi(\omega, q)$ give the eigenfrequencies of collective excitations $\omega = \omega_R + i\omega_I$ and satisfy the following dispersion relation

$$1 + F'_0 \tilde{\chi}_{\text{in}}(\omega, q) = 0. \quad (23)$$

2.1 Boundary condition

For finite nuclei, the dispersion relation (23) has to be augmented by the boundary condition. The boundary condition can be taken as a condition for the balance of the forces on the free nuclear surface

$$\vec{n} \cdot \vec{F}|_S + \vec{n} \cdot \vec{F}_S = 0, \quad (24)$$

where \vec{n} is the unit vector in the normal direction to the nuclear surface S , the internal force \vec{F} is associated with the isovector sound wave and \vec{F}_S is the isovector surface tension force. The internal force \vec{F} is derived by the momentum flux tensor in the nuclear interior and can be evaluating directly using the basic kinetic equation (5), see Appendix B. The isovector surface force \vec{F}_S is due to the isovector polarization at the nuclear surface [43]. Both forces $\vec{n} \cdot \vec{F}|_S$ and $\vec{n} \cdot \vec{F}_S$ in Eq. (24) can be represented in terms of isovector shift of the nuclear surface and the boundary condition (24) takes the final form of the following secular equation the wave number q , see Appendix B,

$$\left[\frac{\bar{\rho}_{\text{eq}}}{4} C_{\text{sym}} + \frac{\mu_F}{3} - \frac{\mu_F}{x^2} \right] j_1(x) + \left[\frac{\mu_F}{x} - \frac{2\rho_{\text{eq}} Q_{\text{sym}}}{3qr_0(1 + \kappa_{NM})} \right] j_1'(x) = 0, \quad (25)$$

where $x = qR_0$, $R_0 = r_0 A^{1/3}$, the parameter μ_F derives the Fermi-surface distortion effect, see Eq. (B 5) in Appendix B, and Q_{sym} is the isovector surface tension coefficient, see Eq. (B 9). In the limit $Q_{\text{sym}} \rightarrow \infty$, the boundary condition (25) leads to the boundary condition $j_1'(x) = 0$ of the Steinwedel-Jensen model [44]. We point out that the secular equation (25) for q has to be solved consistently with the dispersion relation (23) for s .

3 Energy-weighted sums and transport coefficients

The presence of the nonlocal interaction in Eq. (22) leads to some important consequences for the properties of the EWS $m_k(q)$ for isovector mode. Let us introduce the strength function per unit volume

$$S(\omega, q) = \text{Im}\chi(\omega, q)/\pi. \quad (26)$$

The EWS are defined by

$$m_k(q) = \int_0^\infty d(\hbar\omega) (\hbar\omega)^k S(\omega, q). \quad (27)$$

Note that the EWS $m_k(q)$ for odd k can also be evaluated by use the dynamic polarizability $\text{Re}\chi(\omega, q)$. Considering the two limits $\omega \rightarrow 0$ and $\omega \rightarrow \infty$ one can obtain [11]

$$\text{Re}\chi(\omega, q)|_{\omega \rightarrow 0} = 2 [m_{-1}(q) + (\hbar\omega)^2 m_{-3}(q) + \dots] \quad (28)$$

and

$$\text{Re}\chi(\omega, q)|_{\omega \rightarrow \infty} = -\frac{2}{(\hbar\omega)^2} [m_1(q) + (\hbar\omega)^{-2} m_3(q) + \dots]. \quad (29)$$

In the case of cold nucleus $T = 0$ and no relaxation $\tau_1, \tau_2 \rightarrow \infty$, applying Eqs. (28) and (29) to the response function (22) and using Eqs. (A 6)-(A 10), we recover well-known results [11]

$$m_{-1}^{(0)}(q) = \frac{\rho_{\text{eq}}}{2 C_{\text{sym}}}, \quad m_1^{(0)}(q) = \hbar^2 \frac{\rho_{\text{eq}}}{2 m'} q^2, \quad m_3^{(0)}(q) = \hbar^4 \frac{C'_{\text{sym}} \rho_{\text{eq}}}{2 m'^2} q^4. \quad (30)$$

Here, ρ_{eq} is the equilibrium particle density, C_{sym} is the isospin symmetry energy

$$C_{\text{sym}} = b_{\text{sym,vol}} = \frac{2}{3} \epsilon_F (1 + F'_0) \approx 60 \text{ MeV}, \quad (31)$$

where $b_{\text{sym,vol}}$ is the volume part of symmetry energy in the nuclear mass formula [44], ϵ_F is the Fermi energy and m' is the effective mass for isovector mode

$$m' = \frac{m^*}{1 + F'_1/3}, \quad m^* = m(1 + F_1/3) \quad (32)$$

and the upper index "(0)" indicates that the corresponding quantity is taken for $T = 0$ with $\tau_1, \tau_2 \rightarrow \infty$. The renormalized symmetry energy C'_{sym} in Eq. (30) is given by

$$C'_{\text{sym}} = C_{\text{sym}} + 8 \epsilon_F / 15, \quad (33)$$

where the last term on the r.h.s. is due to the Fermi surface distortion effect [36].

In contrast to the isoscalar mode, the isovector EWS $m_1(q)$ of Eq. (30) is model dependent. As can be seen from Eq. (30), the sum $m_1(q)$ includes the enhancement factor (for nuclear matter) [37]

$$1 + \kappa_{NM} = (m/m^*)(1 + F'_1/3), \quad (34)$$

which depends on the nonlocal interaction constant $F'_1 \neq 0$.

In general the inverse EWS m_{-1} derives the static stiffness coefficient. Evaluating the distorted wave function $|\Psi_{\text{ad}}\rangle$ for a static (adiabatic) constrained field $U_{\text{ext}} = \lambda_0 \hat{q} + \lambda_0^* \hat{q}^*$ (see Eq. (1) at $\omega \rightarrow 0$), one can evaluate the corresponding variation of the energy

$$\delta E = \langle \Psi_{\text{ad}} | \hat{H} | \Psi_{\text{ad}} \rangle - \langle \Psi_{\text{eq}} | \hat{H} | \Psi_{\text{eq}} \rangle = \frac{1}{4 m_{-1}^{(0)}} \delta Q^2, \quad (35)$$

where \hat{H} is the non-perturbed Hamiltonian of the nucleus and δQ is the change of the mean value $\langle \hat{q} \rangle$ induced by the distorted wave function. Using Eqs. (30) and (35), we obtain the adiabatic stiffness coefficient (per unit volume) $C_{Q,\text{ad}}$ in the following form

$$C_{Q,\text{ad}} = \frac{\partial^2 \delta E}{\partial \delta Q^2} = \frac{1}{2 m_{-1}^{(0)}} = C_{\text{sym}} / \rho_{\text{eq}}. \quad (36)$$

The cubic sum m_3 is related to the stiffness coefficient $C_{Q,\text{sc}}$ of the scaling approximation [38]. Assuming a scaled form of the perturbed ground state wave function $|\Psi_{\text{sc}}\rangle = e^{-i\lambda_0 \hat{q}} |\Psi_{\text{eq}}\rangle$, one can evaluate the corresponding change of the energy

$$\delta E = \langle \Psi_{\text{sc}} | \hat{H} | \Psi_{\text{sc}} \rangle - \langle \Psi_{\text{eq}} | \hat{H} | \Psi_{\text{eq}} \rangle = \frac{m_3^{(0)}}{4 m_1^{(0)2}} \delta Q^2. \quad (37)$$

Using Eqs. (30), the scaled stiffness coefficient $C_{Q,\text{sc}}$ takes the following form

$$C_{Q,\text{sc}} = \frac{m_3^{(0)}}{2m_1^{(0)2}} = C'_{\text{sym}}/\rho_{\text{eq}}. \quad (38)$$

Note that the constrained stiffness coefficient $C_{Q,\text{ad}}$ of Eq. (36) is not affected by the Fermi-surface distortion since the sum m_{-1} contains the static symmetry energy C_{sym} . This is not case for the stiffness coefficient $C_{Q,\text{sc}}$ because the renormalized C'_{sym} enters the sum m_3 , see Eq. (30). It can be shown [39] that both the sum m_3 and the stiffness coefficient $C_{Q,\text{sc}}$ depend on the Fermi-surface distortions of multipolarity $l \leq 2$.

The EWS m_{-1} , m_1 and m_3 allow obtaining the adiabatic, \tilde{E}_{ad} , and scaled, \tilde{E}_{sc} , average energies (centroid energies) of IVGDR [40]

$$\tilde{E}_{\text{ad}} = \sqrt{\frac{m_1^{(0)}}{m_{-1}^{(0)}}} = \hbar \sqrt{\frac{C_{\text{sym}}}{m'}} q, \quad \tilde{E}_{\text{sc}} = \sqrt{\frac{m_3^{(0)}}{m_1^{(0)}}} = \hbar \sqrt{\frac{C'_{\text{sym}}}{m'}} q. \quad (39)$$

There is a significant difference between the adiabatic energy, \tilde{E}_{ad} , i.e., derived by a static stiffness coefficient C_{sym} , and the scaled one, \tilde{E}_{sc} , associated with the isovector sound in nuclear Fermi liquid. The Fermi-surface distortion effects increase the stiffness coefficient C'_{sym} Eq. (33) and leads to an increase of the centroid energy \tilde{E}_{sc} of the isovector mode.

The low frequency (see Eq. (28)) sum $m_{-3}^{(0)}$ is related to the transport coefficient. Assuming a slow time dependence of the external field $U_{\text{ext}}(t)$ and evaluating the corresponding time dependent wave function $|\Psi(t)\rangle$, one can find the change of the energy in the following form

$$\delta E = \langle \Psi(t) | \hat{H} | \Psi(t) \rangle - \langle \Psi_{\text{eq}} | \hat{H} | \Psi_{\text{eq}} \rangle = \hbar^2 \frac{m_{-3}^{(0)}}{4m_{-1}^{(0)2}} \delta \dot{Q}^2. \quad (40)$$

Using Eq. (40) we obtain the transport (mass) coefficient B_Q as

$$B_Q = \frac{\partial^2 \delta E}{\partial \delta \dot{Q}^2} = \hbar^2 \frac{m_{-3}^{(0)}}{2m_{-1}^{(0)2}}. \quad (41)$$

Using the mass coefficient B_Q and the stiffness coefficient $C_{Q,\text{ad}}$ (see Eq. (36)), the eigenfrequency ω_{macr} and corresponding eigenenergy E_{macr} for the macroscopic eigenvibrations can be derived as

$$\omega_{\text{macr}} = \sqrt{\frac{C_{Q,\text{ad}}}{B_Q}}, \quad E_{\text{macr}} = \hbar \omega_{\text{macr}} = \sqrt{\frac{m_{-1}^{(0)}}{m_{-3}^{(0)}}}. \quad (42)$$

We point out that the sum $m_{-3}^{(0)}$ can not be evaluated within the Landau-Vlasov kinetic approach by the use of the low frequency expansion of Eq. (28). That is because the Fermi liquid gets into Landau-damping regime at $\omega \sim 0$ and special attention must be paid to the low energy region in Eq. (27), see below.

4 Results and Discussions

We will present results of numerical calculations based on the consideration of previous sections. In this work we adopt the value of $r_0 = 1.2 \text{ fm}$ and the effective nucleon mass m^* is taken as $m^* = 0.9m$ which corresponds to the Landau parameter $F_1 = -0.3$. For the isovector interaction parameter F'_0 we have used $F'_0 = 1.41$ to keep a reasonable value of isospin symmetry energy C_{sym} of the order of 60 MeV, see Eq. (31). The interaction parameter F'_1 will be derived and discussed below. For the relaxation parameters in Eq. (12) we use the values of $\alpha_2 = 5.4 \text{ MeV}$ and $\eta = -0.1$ which correspond to the in-medium reduced NN cross sections [16, 17]. For more clear interpretation of some numerical results we will also use the relaxation parameters α_2 and η beyond these well-established values.

4.1 Eigenenergy and enhancement factor in cold nuclei

The interaction parameter F'_1 can be estimated by considering the enhancement factor $1 + \kappa_{NM}$ for the isovector EWS $m_1(q)$, see Eq. (34). Following Ref. [16], we derive the photoabsorption cross section $\sigma_{\text{abs}}(\omega)$ in terms of the strength function $S(\omega, q)$ Eq. (26) as follows

$$\sigma_{\text{abs}}(\omega) = \frac{4\pi^2 e^2}{cq^2 \rho_{\text{eq}}} \frac{NZ}{A} \omega S(\omega, q). \quad (43)$$

In the case of the velocity independent NN -interaction, the cross section $\sigma_{\text{abs}}(\omega)$ (43) is normalized by the ordinary Thomas-Reiche-Kuhn sum rule [45] (see $m_1(q)$ in Eq. (30) for $\kappa_{NM} = 0$)

$$\tilde{m}_{1,\text{TRK}}^{(0)} = \int_0^\infty d(\hbar\omega) \sigma_{\text{abs}}(\omega) = \frac{2\pi^2 \hbar e^2}{mc} \frac{NZ}{A} \quad \text{for } \kappa_{NM} = 0 \quad (44)$$

at $T = 0$ and $\tau_1, \tau_2 \rightarrow \infty$.

Taking into account the velocity dependence of the NN -interaction with $F_1 \neq 0$ and $F'_1 \neq 0$, we note that both the enhancement factor $\kappa_{NM} \neq 0$ in $m_1(q)$ of Eq. (30) and the corresponding correction at the last term of the boundary condition (B 17) affect the sum rule (44). For $\kappa_{NM} \neq 0$, using Eqs. (43), (30) and (B 17), we obtain the following result [39]

$$\tilde{m}_1^{(0)} = \int_0^\infty d(\hbar\omega) \sigma_{\text{abs}}(\omega) = \frac{2\pi^2 \hbar e^2}{mc} \left(\frac{q'_1(A)}{q_0(A)} \right)^2 \frac{NZ}{A} (1 + \kappa_{NM}) \quad \text{for } \kappa_{NM} > 0, \quad (45)$$

at $T = 0$ and $\tau_1, \tau_2 \rightarrow \infty$. Here $q_0(A)$ and $q'_1(A)$ are the lowest roots of Eq. (B 17) for $\kappa_{NM} = 0$ and $\kappa_{NM} \neq 0$, respectively. The value of interaction parameters F'_1 can be now obtained from a fit of the evaluated enhancement factor $\tilde{m}_1^{(0)}/\tilde{m}_{1,\text{TRK}}^{(0)}$ to the experimental data. In this work we have adopted $F'_1 = 1.1$. Our estimate of the enhancement factor is about 10% for light nuclei and increases to 20% for heavy nuclei which is in a good agreement with experimental data [46].

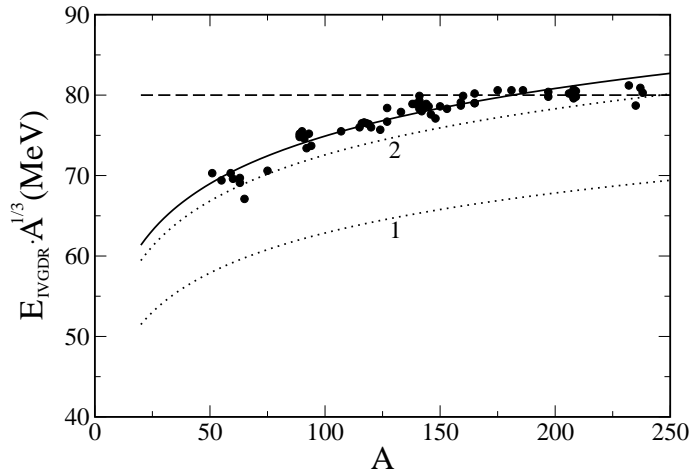


Fig. 1: Dependence of the IVGDR energy on the mass number A : the dotted line 1 is the first-sound regime without Fermi surface distortion (adiabatic approximation, see \tilde{E}_{ad} in the Eq. (39)); the dotted line 2 shows the calculation which takes into account the Fermi surface distortion up to the multipolarity $l = 2$ (scaling approximation, see \tilde{E}_{sc} in the Eq. (39)); the solid line was obtained by use of the dispersion relation (23) and the secular equation (25). The dashed line shows the result within SJ model with commonly used [14, 16, 17] value of the wave number $q = \pi/2R_0$. The solid dots are the experimental data from Ref. [46].

In finite nuclei, both the IVGDR eigenenergy E_{IVGDR} and the EWS m_k are the complicated functions of the mass number A . In contrast to the classical Steinwedel-Jensen model [44], the value $E_{\text{IVGDR}} \cdot A^{1/3}$ is not the constant but increases with A [46]. Within our Fermi liquid approach, the A -dependence of the IVGDR eigenenergy and EWS occurs due to the boundary condition of Eq. (25) on the moving nuclear surface.

In Fig. 1 we show the dependence of the IVGDR energy (multiplied by $A^{1/3}$) on the mass number A . The calculations have been performed for $Q_{\text{sym}} = 10.5$ MeV and $F'_1 = 1.1$. The solid line is the eigenenergy obtained from the dispersion equation (23) augmented by the boundary condition of Eq. (25). Both dotted lines in Fig. 1 have been obtained from the EWS definitions of the centroid energies Eq. (39) (curve 1 for \tilde{E}_{ad} and curve 2 for \tilde{E}_{sc}). The significant upward shift of the scaled energy, \tilde{E}_{sc} , with respect to the constrained one, \tilde{E}_{ad} , is due to the Fermi surface distortions of multipolarity $l \leq 2$ presented in \tilde{E}_{sc} . An additional upward shift of the exact eigenenergy (solid line) is due to the higher multiplicities $l > 2$ of the Fermi surface distortions presented in the dispersion equation (23). As seen in Fig. 1, we reproduce quite well the average behavior of the IVGDR energy E_{IVGDR} . Note that the boundary condition $j'_1(qR_0) = 0$ of the Steinwedel-Jensen model [44] as well as the commonly used wave number $q = \pi/2R_0$ [14, 16, 17] do not describe the A -dependence of the IVGDR energy correctly, see dashed line in Fig. 1.

4.2 First sound regime

We consider the first sound regime as the displacement of the spherically-symmetric Fermi surface without its deformation in momentum space. In the case of the velocity dependent effective NN -interaction with $F'_1 \neq 0$, the first sound eigenvibrations differ from the corresponding ones in the classical Steinwedel-Jensen model [44]. For the sake of simplicity we will consider the longitudinal eigenvibrations assuming $U_{\text{ext}}(t) = 0$ and no relaxation ($\tau_k = \infty$) in Eq. (5). Using Eq. (14) and expansion (instead of Eq. (15))

$$\nu_{\omega, \vec{q}}(\vec{p}) = \sum_{lm} \nu_{lm}(\vec{q}, \omega) Y_{lm}(\hat{p}), \quad (46)$$

we will transform the kinetic equation (5) to the following set of equations

$$\omega \nu_{lm} - v_F q \sum_{l'm'} G'_{l'} \langle lm | \hat{q} \cdot \hat{p} | l'm' \rangle \nu_{l'm'} = 0, \quad (47)$$

where $G'_l = 1 + F'_l/(2l+1)$ and

$$\langle lm | \hat{q} \cdot \hat{p} | l'm' \rangle = \int d\Omega_p Y_{lm}^*(\hat{p}) \cos(\theta_{qp}) Y_{l'm'}(\hat{p}).$$

Using Eq. (19) and assuming $\nu_{lm}|_{l \geq 2} = 0$, we obtain from Eq. (47) the following closed equations for amplitudes ν_{00} and ν_{10} :

$$s \nu_{00} - \frac{1}{\sqrt{3}} G'_1 \nu_{10} = 0, \quad s \nu_{10} - \frac{1}{\sqrt{3}} G'_0 \nu_{00} = 0 \quad (48)$$

and the corresponding dispersion relation

$$\omega = \frac{1}{\sqrt{3}} v_F q \sqrt{G'_0 G'_1}. \quad (49)$$

Finally, using the definition (31) of the symmetry energy C_{sym} , we obtain the eigenenergy E_{first} of the IVGDR in the first sound limit in the following form

$$E_{\text{first}} = \hbar \sqrt{\frac{C_{\text{sym}}}{m} \frac{1 + F'_1/3}{1 + F'_1/3}} q = \hbar \sqrt{\frac{C_{\text{sym}}}{m}} (1 + \kappa_{NM}) q. \quad (50)$$

The energy E_{first} of Eq. (50) differs from the one E_{SJ} of the Steinwedel-Jensen model

$$E_{\text{SJ}} = \hbar \sqrt{\frac{C_{\text{sym}}}{m}} q \quad (51)$$

by the enhancement factor $1 + \kappa_{NM}$. Similar result was reported earlier in Ref. [37].

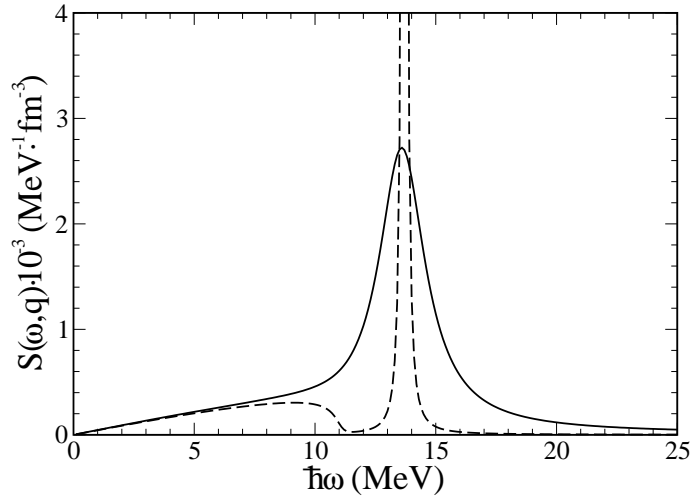


Fig. 2: Strength function $S(\omega, q)$ from Eqs. (22) and (A 10) for $F_1 = -0.3$, $F'_1 = 1.1$, $\eta = -0.1$, $A = 208$. Solid line for $T = 2$ MeV, $\alpha_2 = 5.4$ MeV and dashed line for $T = 0.5$ MeV, $\alpha_2 = 100$ MeV.

4.3 Strength function

Performing the numerical calculations of the response function $\chi(\omega, q)$ (22), one can evaluate the strength function $S(\omega, q)$ (26) and the EWS $m_k(q)$ (27) for finite temperatures $T \neq 0$ and in presence of relaxation. The strength function $S(\omega, q)$ is sensitive to the interaction parameters and to the relaxation properties. Because of $F'_0 > 0$, the IVGDR strength function contains both the sound mode contribution at $s > 1$ and the Landau damping region at $s < 1$. This is illustrated in Fig. 2.

To show the presence of the Landau damping in the IVGDR $S(\omega, q)$ in a transparent way, we have plotted in Fig. 2 the result for the zero-sound regime $\omega_R \tau_2 \gg 1$ at $T = 0.5$ MeV, $\alpha_2 = 100$ MeV (dashed line). The Landau damping appears there as a wide bump on the left side of the narrow sound peak. For high temperature (solid line in Fig. 2), the sound peak becomes wider due to the decrease of the relaxation time (collisional relaxation), see Eq. (12), and due to the collisionless thermal Landau damping, which increases with T , see Ref. [15]. As can be seen from Fig. 2, overlapping of both the sound peak and the Landau damping bump leads to the asymmetry of the IVGDR strength function at high temperatures. This feature of the IVGDR strength function is observed experimentally [7]. Note that the IVGDR width, which can be derived from Fig. 2, represents a collisional part of total width only and this one is significantly smaller the experimental width of Ref. [7]. Additional part of the IVGDR width is caused by the fragmentation mechanism and we will take into account this fact below in Fig. 8.

Sensitivity of the strength function $S(\omega, q)$ to the interaction parameter F'_1 is demonstrated in Fig. 3). The inclusion of the nonlocal interaction $F'_1 \neq 0$ increases the isovector

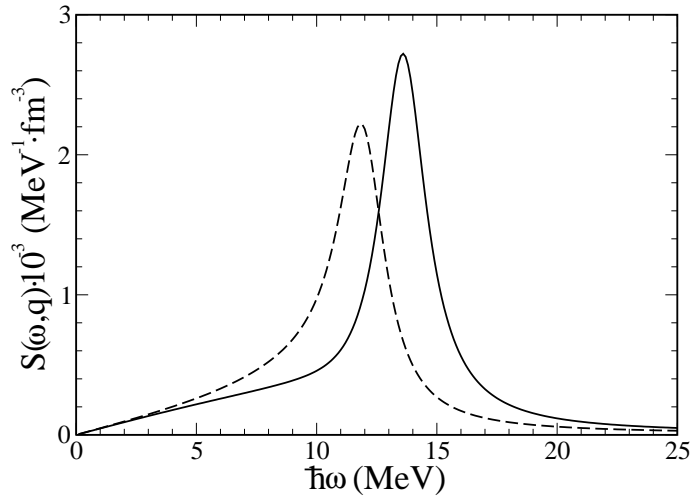


Fig. 3: Strength function $S(\omega, q)$ from Eqs. (22) and (A 10) for $T = 2$ MeV, $\alpha_2 = 5.4$ MeV, $F_1 = -0.3$, $\eta = -0.1$, $A = 208$. Solid line for $F'_1 = 1.1$, and dashed line for $F'_1 = 0$.

stiffness coefficient and shifts the IVGDR to the higher energy. Moreover, since the interaction parameter F'_1 enters the enhancement factor $1 + \kappa_{NM}$ of Eq. (34), the photoabsorption cross section $\sigma_{\text{abs}}(\omega)$ (43) grows with $F'_1 > 0$.

Presence of the $1/\tau_1$ term in the collision integral in Eq. (11) increases the width of the IVGDR resonance and does not much affect its energy centroid. However, this term influences strongly the zero- to first-sound transition for the isovector mode [17, 18]. In general, a decrease of the collisional relaxation time τ_2 leads to the fast damping of the Fermi surface distortions and thereby to the zero- to first-sound transition. This is demonstrated in Fig. 4.

The solid line in Fig. 4 shows the numerical result for the photoabsorption cross section $\sigma_{\text{abs}}(\omega)$ from Eq. (43) for long relaxation time regime (zero-sound regime, $\alpha_2 = 5.4$ MeV) at $\eta = 1$, i.e., $\tau_1 = \infty$. To show the zero- to first-sound transition, we have plotted in Fig. 4 (dashed line) the cross section $\sigma_{\text{abs}}(\omega)$ which is obtained in the first sound regime $\omega_R \tau_2 \ll 1$ at $\alpha_2 = 0.1$ MeV. This transition happens as a shift of the resonance energy to the energy of the first sound eigenmode, E_{first} , given by Eq. (50)

$$E_{\text{first}} \approx 17.5 \text{ MeV} \quad \text{for} \quad A = 208.$$

This value of E_{first} significantly exceeds the SJ estimate $E_{\text{SJ}} \approx 14.6$ MeV for $A = 208$ obtained at the boundary condition $j'_1(qR_0) = 0$ [44], see also Eqs. (50) and (51).

The behavior of the photoabsorption cross section $\sigma_{\text{abs}}(\omega)$ is essentially different for the case where the relaxation time τ_2 exceeds significantly the relaxation time τ_1 . In this case the relaxation of the relative proton-neutron motion is faster than the zero- to first-sound transition and the first-sound peak of the IVGDR disappears. This effect is shown in Fig. 5 where the dashed line was obtained at $\eta = -1$, i.e., $\tau_2 = 2\tau_1$.

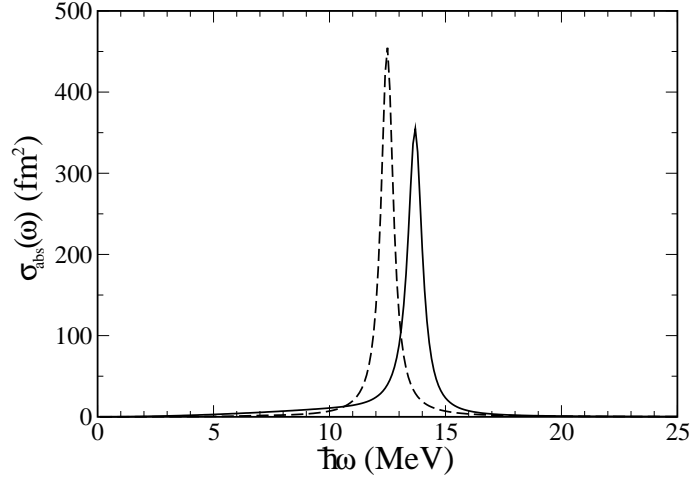


Fig. 4: Photoabsorption cross section $\sigma_{\text{abs}}(\omega)$ from Eqs. (22), (43) and (A 10) for $T = 2$ MeV, $A = 208$, $\eta = 1$, $F_1 = 0$, $F'_1 = 0$. Solid line for $\alpha_2 = 5.4$ MeV (zero-sound regime) and dashed line for $\alpha_2 = 0.1$ MeV (first-sound regime).

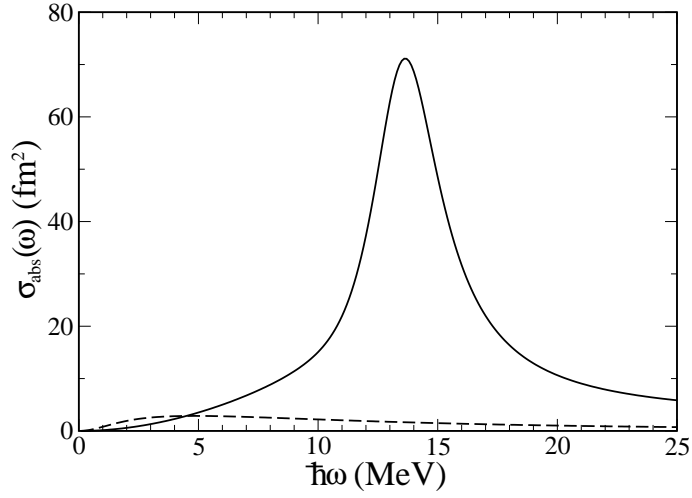


Fig. 5: The same as Fig. 4, but for $\eta = -1$.

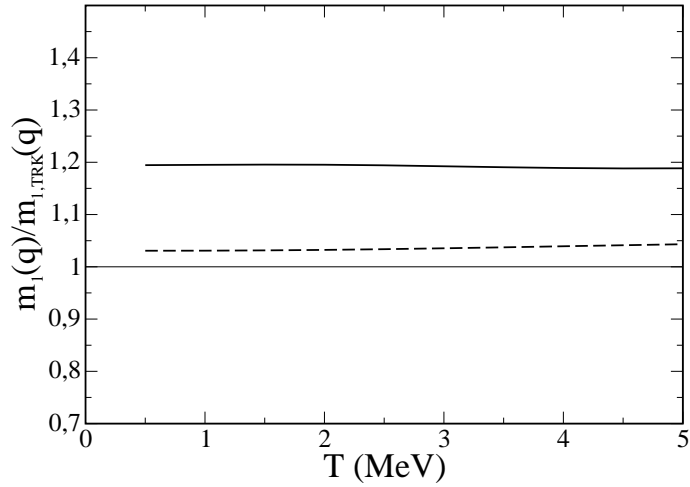


Fig. 6: Temperature dependence of the "model independent" EWS $m_1(q)$ obtained from Eqs. (27), (26) and (22) normalized to the TRK sum rule $m_{1,TRK}(q)$. The calculations performed for $A = 208$ using $\alpha_2 = 5.4$ MeV, $\eta = -0.1$ and $F_1 = -0.3$. Solid line for $F'_1 = 1.1$ and dashed line for $F'_1 = 0$.

4.4 Energy-weighted sums and centroid energies

We have studied the temperature behavior of the "model independent" EWS $m_1(q)$ and the enhancement factor $m_1(q)/m_{1,TRK}(q)$, where, see Eq. (27),

$$m_{1,TRK}(q) = \int_0^\infty d(\hbar\omega) \hbar\omega S(\omega, q) \quad \text{for} \quad F_1 = F'_1 = 0.$$

For non-zero temperatures and in presence of the relaxation, the energy-weighted sum $m_1(q)$ has been evaluated using the definition (27) and the response function $\chi(\omega, q)$ from Eq. (22). In Fig. 6, we have plotted the ratio $m_1(q)/m_{1,TRK}(q)$ as a function of temperature T .

As can be seen from Fig. 6, the enhancement factor is only slightly sensitive to the temperature variation. For the nucleus ^{208}Pb , we have from Fig. 6 the following estimate

$$m_1(q)/m_{1,TRK}(q) \approx 1.2.$$

In Fig. 7, we compare the centroid energies $\tilde{E}_1 = \sqrt{m_1/m_{-1}}$ and $\tilde{E}_3 = \sqrt{m_3/m_1}$ evaluated using Eq. (27), and the IVGDR eigenenergy E_R obtained from Eqs. (23) and (25).

The significant upward shift of energy \tilde{E}_3 curve with respect to \tilde{E}_1 is due to the Fermi surface distortion effect. The cubic sum m_3 which enters the expression for \tilde{E}_3 is associated with the scaling transformation and the quadrupole distortion of the Fermi surface [42]. On the other hand, the distortion of the Fermi surface causes an additional contribution to the

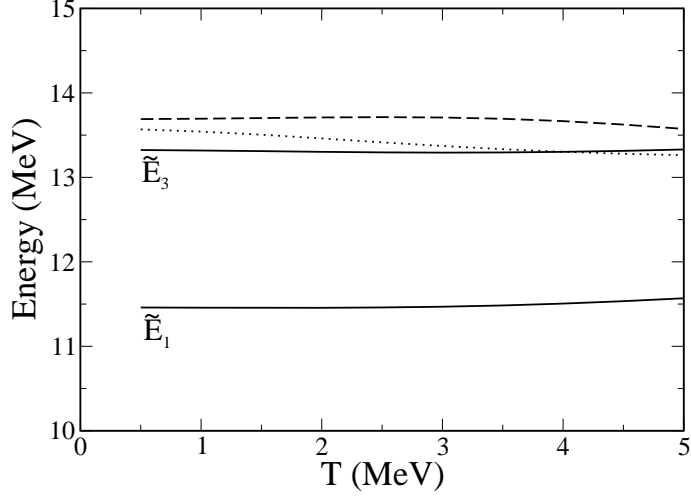


Fig. 7: Temperature dependence of the centroid energies $\tilde{E}_1 = \sqrt{m_1/m_{-1}}$ (solid line) and $\tilde{E}_3 = \sqrt{m_3/m_1}$ (solid line) obtained from Eq. (27), and the IVGDR eigenenergy E_R (dashed line) obtained from Eqs. (23) and (25). The calculations were performed for $A = 208$ using $\alpha_2 = 5.4$ MeV, $\eta = 2/3$, $m^*/m = 0.9$ and $F'_1 = 1.1$. Dotted line shows the classical energy E_{macr} from Eq. (42).

stiffness coefficient (see also Eqs. (33) and (38)) providing a growth of the centroid energy \tilde{E}_3 . In contrast to \tilde{E}_3 , the centroid energy \tilde{E}_1 is derived by the inverse sum m_{-1} , where the contribution from the Fermi-surface distortion effects is negligible, see also Eq. (36). Note that the presence of the Fermi-surface distortion effects increases also the resonance energy E_R because the dispersion equation (23) includes all multipolarities of the Fermi surface distortion. A small decrease of the energy E_R with growing temperature T in Fig. 7 is due to the fact that the Fermi-surface distortion effects become weaker for higher temperatures. In contrast to \tilde{E}_3 , which is nearly temperature independent, the centroid energy \tilde{E}_1 increases slightly with T . That is because a growth of resonance width with temperature leads to a decrease of the inverse sum m_{-1} in agreement with its definition of Eq. (27).

In Fig. 7, the dotted line shows the behavior of the eigenenergy E_{macr} for the macroscopic eigenvibrations given by Eq. (42). To provide the convergency of the integral in Eq. (27) for the inverse sum m_{-3} in Eq. (42), we have introduced the cut off parameter E_{cut} in Eq. (27). Namely, we have used

$$m_{-3}(q) = \int_{E_{\text{cut}}}^{\infty} d(\hbar\omega) (\hbar\omega)^{-3} S(\omega, q). \quad (52)$$

The cut off energy E_{cut} was derived from the requirement that the Landau-damping region $s < 1$ should be removed from the EWS m_k since m_k must be related to a given sound eigenmode only. Note that the energy interval $[E_{\text{cut}}, \infty]$ must be large enough to provide a reasonable exhaustion ($\gtrsim 90\%$) of sum rule (45). A good agreement of E_{macr} with the

IVGDR eigenenergy allows one to conclude that the cut off procedure in Eq. (52) can also be used for the consistent evaluation of the mass coefficient B_Q of Eq. (41).

Finally, it should be noted that there is the limiting temperature T_{lim} for the IVGDR existence. The limiting temperature can be deduced from the decomposition of the $\tilde{\chi}_{\text{in}}(\omega, q)$ of Eq. (A 6) in powers of $1/s$ for the first sound limit (high temperature regime) at $|s| \gg 1$. The presence of the finite relaxation time τ_1 caused by the collisions for the counterstreaming neutron and proton flows leads to the additional temperature dependence of the eigenfrequency which is specific for the isovector mode only. The eigenfrequency of the isovector first sound decreases with temperature and disappears at the limiting temperature T_{lim} . The magnitude of the limiting temperature T_{lim} for the IVGDR depends significantly on the relaxation parameter η . For heavy nuclei, the numerical estimate [17] provides the value of $T_{\text{lim}} \simeq 7$ MeV.

4.5 Damping and spreading width of IVGDR

The EWS $m_k(q)$ can be used to analyze the spreading of the strength function. We will use the following definition of the spreading width [11, 47]

$$\gamma(q) = \frac{m_1(q)}{m_0(q)} - \frac{m_0(q)}{m_{-1}(q)}. \quad (53)$$

In the case of a Lorentzian shape of the photoabsorption cross section

$$\sigma_{\text{abs}}(\omega) = \frac{\sigma_0 E^2 \Gamma^2}{(E - E_0)^2 + E^2 \Gamma^2}, \quad E = \hbar\omega, \quad (54)$$

one can find from Eqs. (43), (27) and (53) the relationship between the quantities \tilde{E}_1 and $\gamma(q)$ and the resonance characteristics E_0 and Γ

$$E_0 = \tilde{E}_1, \quad \gamma(q) = \frac{2}{\pi} \Gamma (1 + O(\Gamma/E_0)) \quad \text{for} \quad \Gamma/E_0 \ll 1. \quad (55)$$

In the case of small damped collective vibrations ($\Gamma \ll E_0$), the resonance collisional width Γ can also be evaluated from the dispersion equation (23). The solution of this equation

$$\omega = \omega_R + i\omega_I, \quad (56)$$

defines the energy of the giant resonance $E_R = \hbar\omega_R$ and its collisional width $\Gamma_{\text{col}} = -2\hbar\omega_I$. In the application of the kinetic approach, in particular, of the collision integral $\delta\text{St}[f]$ to the finite nucleus, the difficulty is the derivation of the NN cross sections σ_{av} and σ_- in Eq. (13) which become \vec{r} -dependent in nuclear interior. The above mentioned (see comment to Eq. (13)) vacuum and in-medium values of the NN cross sections and thereby relaxation parameter α_2 in Eqs. (12) and (13) give the lower and upper theoretical limits. The \vec{r} -dependence of the relaxation parameter α_2 can be taken into account phenomenologically

by introduce of the effective relaxation parameter α_{eff} as following

$$\frac{1}{\alpha_{\text{eff}}} = \frac{\int d\vec{r} \rho_{\text{eq}}(\vec{r}) / \alpha_2(\vec{r})}{\int d\vec{r} \rho_{\text{eq}}(\vec{r})}, \quad (57)$$

where $\alpha(\vec{r})$ is parametrized by

$$\alpha_2(\vec{r}) = \alpha_{\text{vac}} + \frac{\rho_{\text{eq}}(\vec{r})}{\rho_{\text{eq}}(0)} (\alpha_{\text{bulk}} - \alpha_{\text{vac}}),$$

with $\alpha_{\text{vac}} = 2.3$, MeV and $\alpha_{\text{bulk}} = 5.4$, MeV, see comments after Eq. (13). The equilibrium particle density $\rho_{\text{eq}}(\vec{r})$ is derived as

$$\rho_{\text{eq}}(\vec{r}) = \rho_0 \left/ \left[1 + \exp \frac{r - R_0}{a} \right] \right.$$

where $\rho_0 = (4\pi r_0^3/3)^{-1}$ and $a = 0.6$ fm. Below we will replace relaxation parameter α_2 in Eq. (12) by the effective one α_{eff} .

Besides the collisional width Γ_{col} , the experimentally observable width of the IVGDR includes the fragmentation width. Within the semiclassical kinetic theory, this mechanism of resonance spreading can be considered as an additional dissipation due to the single particle scattering on the moving surface of the nucleus (one-body dissipation [48, 49, 50]). Instead of τ_2 , we will use the effective relaxation time τ_{eff} which contains the contribution from both two-body and one-body dissipations. Namely, see also Refs. [17, 33],

$$\frac{1}{\tau_{\text{eff}}} = \frac{1}{\tau_2} + \frac{1}{\tau_{\text{wall}}}. \quad (58)$$

We will use the one-body relaxation time τ_{wall} in the following form [33]

$$\tau_{\text{wall}} = \frac{2R_0}{\bar{v}} \xi, \quad \bar{v} = \frac{3v_F}{4} \left[1 + \frac{\pi^2}{6} \left(\frac{T}{\epsilon_F} \right)^2 \right]. \quad (59)$$

The parameter ξ in Eq. (59) depends on the model of the one-body dissipation [48, 49, 50, 51]. We consider ξ as a free parameter which is determined from a fit of the total IVGDR width Γ to the experimental data at zero temperature $T = 0$.

In Fig. 8 we have plotted the temperature dependence of the width Γ derived from the EWS by use of Eq. (55) (solid line 1) and from the dispersion relation applying Eqs. (23), (56), (57) and (58) (solid line 2). The numerical calculations were performed for the nucleus ^{120}Sn where the experimental data are known for a wide range of temperatures $T = 0 \div 3$ MeV [5, 6, 7].

We point out that considering the experimental data we assume that the nuclear excitation energy E^* is related to the nuclear temperature T by the Fermi-gas formula

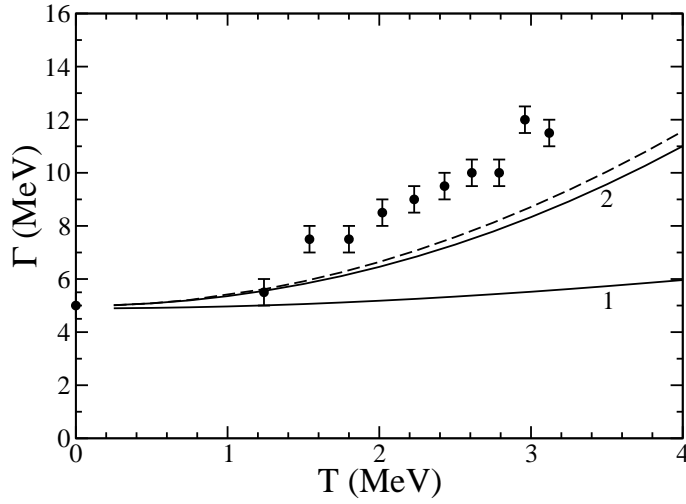


Fig. 8: Temperature dependence of the IVGDR width Γ : the solid line 1 was obtained from the EWS definition given by Eqs. (53) and (55); the solid line 2 and the dashed line were obtained from the dispersion equation (23). The calculations were performed for the nucleus ^{120}Sn using α_{eff} from Eq. (57) and $\eta = -0.1$. Solid line 2 for $F_1 = -0.3$, $F'_1 = 1.1$ and dashed line for $F_1 = F'_1 = 0$.

$T = \sqrt{E^*/a}$ where a is level density parameter. This fact leads to some uncertainty because the derivation of parameter a is model dependent. However, within the kinetic theory, the use of the macrocanonical ensemble, and thereby temperature T , is preferable because the ensemble smearing is assumed at the derivation of basic collisional kinetic equation.

In Fig. 8, the total resonance width Γ grows with temperature mainly due to the temperature dependence of the collisional relaxation time τ_2 , see Eq. (12). As can be seen from Eq. (59), the temperature dependence of the one-body dissipation is too weak to be responsible for the fast increase of the IVGDR width. The result in Fig. 8 shows also that the collisional width does not give a good description of the observable growing of the IVGDR width with T . An additional increase of the IVGDR width with temperature can be achieved taking into consideration the coupling of the IVGDR to thermal shape fluctuations [25] which do not present in our approach. Note that a decrease of the in-medium NN cross section leads to an increase of the temperature dependence of the IVGDR width [16] also.

For low temperatures, the total resonance width Γ derived by the EWS (solid line 1 in Fig. 8) is close to the one obtained from the dispersion relation (solid line 2 in Fig. 8). That is due to the fact that the evaluated photoabsorption cross section approaches to the Lorentzian shape in this case. For higher temperatures, the assumption $\Gamma/E_0 \ll 1$ in Eq. (55) is not fulfilled and the expressions (53) and (55) for the derivation of Γ can not be correctly used.

5 Summary and Conclusions

A goal of this work is the derivation of the macroscopic characteristics of isovector modes in a two-component Fermi-liquid drop from our knowledge of the nuclear IVGDR. Starting from the collisional kinetic equation (5), we have derived the strength function and the energy weighted sums m_k for the isovector excitations in the heated nuclear matter and the finite nuclei. An important ingredient of our consideration is the inclusion of the velocity dependent NN -interaction for both isovector and isoscalar channels. Our consideration is valid for an arbitrary collision parameter α_l in Eq. (12) and can be used, particularly, for the transition region from the zero sound- to first sound (hydrodynamic) regime in nuclear Fermi-liquid.

We have adopted a Fermi liquid drop model with two essential features: (i) The linearized kinetic equation is applied to the nuclear interior, where the relatively small oscillations of the particle density take place; (ii) The dynamics in the surface layer of the nucleus is described by means of the macroscopic boundary condition which is taken as a condition for the balance of the forces on the free nuclear surface.

Due to the consistent solution of the dispersion relation (23) and secular equation (25), our model provides a satisfactory description of the A -dependence of the eigenenergy of the IVGDR, see Fig. 1. In contrast to earlier consideration of Ref. [24] performed within the scaling approximation, we take into account all multipolarity l of the Fermi surface distortion. Moreover, the value of Landau's interaction parameter $F'_1 \approx 1.1$ has been derived from a fit of the evaluated EWS enhancement factor to the experimental data. Note that we do not use a concept of the effective NN -interaction within the surface layer of the nucleus applying instead the relevant boundary condition and avoiding thereby the uncertainty in the derivation of effective interaction in the region of strong particle density inhomogeneity.

The present study has shown the following:

1. The Landau damping occurs in the isovector strength function $S(\omega, q)$ at low temperatures as a wide bump on the left side of the narrow sound peak (see Fig. 2). For high temperature, the overlapping of both the sound peak and the Landau damping bump leads to the asymmetry of the IVGDR strength function $S(\omega, q)$. A similar asymmetry of the IVGDR strength function is also observed experimentally at non-zero temperatures [7].
2. The isovector EWS shows only minor temperature dependence. In particular, the "model independent" EWS $m_1(q)$ and the corresponding enhancement factor $m_1(q)/m_{1,TRK}(q)$ are practically constant in the interval of temperature $T = 0 \div 5$ MeV, see Fig. 6.
3. The "model independent" EWS $m_1(q)$ and the enhancement factor $m_1(q)/m_{1,TRK}(q)$ are slightly sensitive to the relaxation (damping) processes. Note that the corresponding problem can not be accurately considered within the standard quantum mechanics because of non-Hermite Hamiltonian in this case.

4. The inclusion of the nonlocal interaction $F'_1 \neq 0$ increases the isovector stiffness coefficient and shifts the IVGDR energy to the higher values. The lowest order EWS $m_{-1}(q)$, $m_1(q)$ and $m_3(q)$ derive the adiabatic, \tilde{E}_1 , and scaled, \tilde{E}_3 , energy centroids. The centroid energy \tilde{E}_1 is close to the classical result of the Steinwedel-Jensen model and differs from the first sound limit by the enhancement factor $1 + \kappa_{NM}$, see Eq. (39). The Fermi distortion effects do not contribute into the centroid energy \tilde{E}_1 . In contrast, the scaling energy \tilde{E}_3 is associated with the quadrupole distortion of the Fermi surface and exceeds significantly the centroid energy \tilde{E}_1 .
5. The often used classical derivation of the IVGDR eigenenergy E_{macr} Eq. (42) through the isovector mass coefficient B_Q has to be revised in the case of nuclear Fermi liquid. That is because the inverse sum m_{-3} , which enters the mass coefficient B_Q , can not be directly evaluated within the kinetic theory due to the Landau-damping region at $\omega \sim 0$. To provide the convergency of the sum m_{-3} , we have proposed the cut-off procedure introducing an appropriate cut off parameter E_{cut} into the definition of m_{-3} in Eq. (27). Due to this procedure we have achieved a good agreement of E_{macr} with the adiabatic centroid energy \tilde{E}_1 . In general, such kind of cut-off procedure can also be used for the evaluation of the transport coefficients within the Fermi-liquid theory.
6. The inclusion into the collision integral of the relaxation of $l = 1$ component (term $\sim 1/\tau_1$ in Eq. (11)) influences strongly the zero- to first-sound transition for the isovector mode. In particular, in the case of $\tau_2 > \tau_1$, the relaxation of the relative proton-neutron motion is too fast and the short-relaxation limit $\alpha_2 \rightarrow 0$ does not provide the zero- to first-sound transition (see disappearance of the first-sound peak in Fig. 5). A similar phenomenon was earlier discussed in Refs. [17, 18] where the velocity independent Landau's interaction with $F'_1 = 0$ has been used.
7. Our analysis of the IVGDR width performed within the kinetic theory shows that the collisional and one-body damping does not reproduce the fast increase of the IVGDR width with temperature. The additional mechanisms of damping, e.g., the coupling of the IVGDR to thermal shape fluctuations [25], have to be involved to improve the agreement of the temperature dependence of the IVGDR width with the experimental data.

Finally, we would like to note that the semiclassical kinetic approach, used in this article, is highly convenient for a study of the averaged properties of the nuclear dynamics. This approach provides an information on the macroscopic characteristics without a detailed knowledge of the wave function of the nucleus. An important advantage of the kinetic theory is that the temperature and the relaxation effects enter the equation of motion directly. Many results can be here presented in a clear and transparent form. Here, the claim is to describe the general features of collective excitations, such as the A -dependence of the IVGDR energy, in a systematic way ignoring many quantum effects, e.g., the shell structure effects. In particular, the kinetic approach allows us to compare the results of standard

liquid drop model [44] with the Fermi liquid drop one where the dynamic distortions of Fermi surface are taken into consideration.

Appendix A: Internal response function

Using Eq. (17), we will evaluate the averaged amplitude $\tilde{\nu}_0$. Taking into account Eq. (19), we will reduce Eq. (17) to the following coupled equations

$$\nu_0(p) + F'_0 Q_{00}(z) \tilde{\nu}_0 + \frac{F'_1}{3} Q_{10}(z) \tilde{\nu}_1 - \lambda_0 Q_{00}(z) = i\gamma \nu_0(p) \frac{1}{z} [1 - Q_{00}(z)] - i\eta \gamma \nu_1(p) Q_{00}(z), \quad (\text{A } 1)$$

$$\nu_1(p) + 3F'_0 Q_{10}(z) \tilde{\nu}_0 + F'_1 Q_{11}(z) \tilde{\nu}_1 - 3\lambda_0 Q_{10}(z) = -3i\gamma \nu_0(p) Q_{00}(z) - 3i\eta \gamma \nu_1(p) Q_{10}(z), \quad (\text{A } 2)$$

where we have used $Q_{10}(z) = z Q_{00}(z)$. Solving Eqs. (A 1) and (A 2) we obtain

$$\nu_0(p) = \frac{z \chi_0(z)}{sD(z) + i\gamma \chi_0(z)} \lambda_0 - F'_0 \frac{z \chi_0(z)}{sD(z) + i\gamma \chi_0(z)} \tilde{\nu}_0 - \frac{F'_1}{3} \frac{z \chi_0(z)(z - i\eta \gamma)}{sD(z) + i\gamma \chi_0(z)} \tilde{\nu}_1 \quad (\text{A } 3)$$

and

$$\begin{aligned} \nu_1(p) = & \frac{3z s \chi_0(z)}{sD(z) + i\gamma \chi_0(z)} \lambda_0 - F'_0 \frac{3z s \chi_0(z)}{sD(z) + i\gamma \chi_0(z)} \tilde{\nu}_0 \\ & - F'_1 \frac{[z^2 \chi_0(z) + 1/3] [sD(z) + i\gamma \chi_0(z)] - i\gamma z (z - i\eta \gamma) \chi_0^2(z)}{D(z) [sD(z) + i\gamma \chi_0(z)]} \tilde{\nu}_1, \end{aligned} \quad (\text{A } 4)$$

where $\chi_0(z) = Q_{00}(z)$,

$$D(z) = 1 + 3i\eta \gamma z \chi_0(z)$$

and the relation $Q_{11}(z) = z^2 \chi_0(z) + 1/3$ has been used. Multiplying both Eqs. (A 3) and (A 4) by $-[g/(2\pi\hbar)^3 N_T] (\partial f_{\text{eq}}/\partial \epsilon_p)$, integrating over \vec{p} and using Eqs. (8) and (18), we obtain two closed equations for amplitudes $\tilde{\nu}_0$ and $\tilde{\nu}_1$. Solving then these equations with respect to $\tilde{\nu}_0$, we obtain

$$\tilde{\nu}_0 = \frac{\tilde{\chi}_{\text{in}}(\omega, q)}{1 + F'_0 \tilde{\chi}_{\text{in}}(\omega)} \lambda_0. \quad (\text{A } 5)$$

Here $\tilde{\chi}_{\text{in}}(\omega, q)$ is the internal response function

$$\tilde{\chi}_{\text{in}}(\omega, q) = \tilde{\chi}_{\text{in},0}(\omega, q) - F'_1 \frac{\tilde{\chi}_s^{(1)}(\omega, q) \tilde{\chi}_\eta^{(1)}(\omega, q)}{1 + F'_1 \tilde{\chi}^{(2)}(\omega, q)}, \quad (\text{A } 6)$$

where

$$\tilde{\chi}_{\text{in},0}(\omega, q) = -\frac{1}{N_T} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \frac{z \chi_0(z)}{sD(z) + i\gamma \chi_0(z)}, \quad (\text{A } 7)$$

$$\tilde{\chi}_s^{(1)}(\omega, q) = -\frac{1}{N_T} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \frac{z s \chi_0(z)}{sD(z) + i\gamma \chi_0(z)}, \quad (\text{A } 8)$$

$$\tilde{\chi}_\eta^{(1)}(\omega, q) = -\frac{1}{N_T} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \frac{z(z - i\eta\gamma)\chi_0(z)}{sD(z) + i\gamma\chi_0(z)} \quad (\text{A } 9)$$

and

$$\begin{aligned} \tilde{\chi}^{(2)}(\omega, q) = & -\frac{1}{N_T} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} \frac{\partial f_{\text{eq}}(\epsilon_p)}{\partial \epsilon_p} \\ & \times \frac{[z^2\chi_0(z) + 1/3] [sD(z) + i\gamma\chi_0(z)] - i\gamma z(z - i\eta\gamma)\chi_0^2(z)}{D(z) [sD(z) + i\gamma\chi_0(z)]}. \end{aligned} \quad (\text{A } 10)$$

Appendix B: Boundary condition

In this appendix we are going to determine the boundary condition from the balance of the forces on the free nuclear surface given by Eq. (24). The internal force \vec{F} in Eq. (24) is related to the momentum flux tensor $\Pi_{\alpha\beta}$ in the nuclear interior

$$F_\alpha = n_\beta \Pi_{\alpha\beta}. \quad (\text{B } 1)$$

The momentum flux tensor $\Pi_{\alpha\beta}$ can be evaluated using the basic kinetic equation (5). Taking the 1-st \vec{p} -moment of Eq. (5) one can obtain the following expression for the momentum flux tensor $\Pi_{\alpha\beta}$ [41, 42]

$$\Pi_{\alpha\beta} = \delta P \delta_{\alpha\beta} + \delta\sigma_{\alpha\beta}, \quad (\text{B } 2)$$

where δP is the pressure caused by the isovector sound wave

$$\delta P = \frac{1}{3m} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} p^2 \delta f_-(\vec{r}, \vec{p}; t) + \frac{F'_0}{N_F} \bar{\rho}_{\text{eq}} \delta \rho_-(\vec{r}, t) = C_{\text{sym}} \delta \rho_-(\vec{r}, t) \quad (\text{B } 3)$$

and $\delta\sigma_{\alpha\beta}$ is the pressure tensor due to the Fermi surface distortion effect

$$\begin{aligned} \delta\sigma_{\alpha\beta} &= \frac{1}{3m} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} (3p_\alpha p_\beta - p^2 \delta_{\alpha\beta}) \delta f(\vec{r}, \vec{p}; t) \\ &= \mu_F \left(\nabla_\alpha \chi_\beta + \nabla_\beta \chi_\alpha - \frac{2}{3} \vec{\nabla} \cdot \vec{\chi} \delta_{\alpha\beta} \right). \end{aligned} \quad (\text{B } 4)$$

Here $\bar{\rho}_{\text{eq}} = (\rho_{\text{eq},n} + \rho_{\text{eq},p})/2$,

$$\mu_F = \frac{3}{2} \epsilon_F \rho_{\text{eq}} \frac{s_R^2}{1 + F'_1/3} \left[1 - \frac{(1 + F'_0)(1 + F'_1/3)}{3 s_R^2} \right], \quad s_R = \frac{\omega_R}{v_F q}, \quad (\text{B } 5)$$

and $\vec{\chi}$ is the displacement field related to the velocity field \vec{u} . Namely,

$$\frac{\partial}{\partial t} \vec{\chi}(\vec{r}, t) = - (1 + \kappa) \vec{u}(\vec{r}, t), \quad \vec{u}(\vec{r}, t) = \frac{1}{\rho_{\text{eq}}} \int \frac{gd\vec{p}}{(2\pi\hbar)^3} \frac{\vec{p}}{m} \delta f_-(\vec{r}, \vec{p}, t). \quad (\text{B } 6)$$

Using Eqs. (B 1) - (B 4), we obtain

$$\vec{n} \cdot \vec{F}|_S = \left[\left(C_{\text{sym}} \bar{\rho}_{\text{eq}} - \frac{2}{3} \mu_F \right) \vec{\nabla} \cdot \vec{\chi} + 2\mu_F \frac{\partial}{\partial r} (\vec{n} \cdot \vec{\chi}) \right]_{r=R_0}. \quad (\text{B } 7)$$

To evaluate the isovector surface tension force \vec{F}_S we will consider the variation $\delta E_{S,\text{sym}}$ of the surface symmetry energy caused by the isovector polarization at the nuclear surface [43]

$$\delta E_{S,\text{sym}} = \frac{1}{3} \rho_{\text{eq}} r_0 Q_{\text{sym}} \int dS \xi^2, \quad (\text{B } 8)$$

where Q_{sym} is the coefficient related to the volume, $b_{\text{sym},\text{vol}}$, and surface, $b_{\text{sym},\text{surf}}$, terms entering to the symmetry energy E_{sym} in the mass formula

$$Q_{\text{sym}} = \frac{9}{8} \frac{b_{\text{sym},\text{vol}}^2}{b_{\text{sym},\text{surf}}}, \quad E_{\text{sym}} = \frac{1}{2} \frac{(N - Z)^2}{A} (b_{\text{sym},\text{vol}} - b_{\text{sym},\text{surf}} A^{-1/3}). \quad (\text{B } 9)$$

In Eq. (B 8), the parameter ξ is the dynamic isovector shift of neutron-proton spheres in units of r_0

$$\xi = \frac{1}{r_0} [R_n(t) - R_p(t)] = \frac{\delta R_1(t)}{r_0}, \quad (\text{B } 10)$$

where

$$\delta R_1(t) = R_0 \alpha_S(t) Y_{10}(\hat{r}). \quad (\text{B } 11)$$

The amplitude $\alpha_S(t)$ of the isovector shift of the nuclear surface is connected with the displacement field $\vec{\chi}$. To establish this connection we note that, for the case of sharp nuclear surface, the isovector displacement field $\vec{\chi}$ is given by [44]

$$\vec{\chi}(\vec{r}, t) = \alpha_1(t) \frac{1}{q^2} \vec{\nabla}_{\vec{r}} [j_1(qr) Y_{10}(\hat{r})]. \quad (\text{B } 12)$$

The boundary condition for the normal component of the velocity field reads

$$\vec{n} \cdot \vec{u}|_S = \frac{\partial}{\partial t} \delta R_1(t). \quad (\text{B } 13)$$

Using Eqs. (B 6), (B 11) and (B 13)

$$\alpha_S(t) = - \alpha_1(t) \frac{j_1'(x)}{x(1 + \kappa)}, \quad x = qR_0. \quad (\text{B } 14)$$

The variation $\delta E_{S,\text{sym}}$ of the surface energy derives the surface pressure

$$\delta P_S = \frac{\partial}{\partial \delta R_1} \frac{\delta E_S}{\delta S} = \frac{8}{3} \frac{\rho_{\text{eq}}}{r_0} Q_{\text{sym}} \delta R_1(t). \quad (\text{B } 15)$$

Taking into account Eqs. (B 11), (B 14) and (B 15)

$$\vec{n} \cdot \vec{F}_S = - \delta P_S = \frac{8}{3} \frac{\rho_{\text{eq}}}{qr_0(1 + \kappa)} Q_{\text{sym}} \alpha_1(t) Y_{10}(\hat{r}). \quad (\text{B } 16)$$

Inserting Eqs. (B 7) and (B 16) into Eq. (24) and using Eq. (B 12), we obtain the following secular equation

$$\left[-\frac{1}{2} C_{\text{sym}} \bar{\rho}_{\text{eq}} - \frac{2}{3} \mu_F + \frac{2}{x^2} \mu_F \right] j_1(x) + \left[-\frac{2}{x} \mu_F + \frac{4}{3} \frac{\rho_{\text{eq}}}{qr_0(1 + \kappa)} Q_{\text{sym}} \right] j_1'(x) = 0. \quad (\text{B } 17)$$

References

- [1] S. Shlomo and V.M. Kolomietz, Rep. Progr. Phys. **68**, 1 (2005).
- [2] K.A. Snover, Ann. Rev. Nucl. Part. Sci. **36**, 545 (1986).
- [3] J.J. Gaardøje, Ann. Rev. Nucl. Part. Sci. **42**, 483 (1992).
- [4] D. Pierroutsakou et al., Nucl. Phys. **A600**, 131 (1996).
- [5] E. Ramakrishnan et al., Phys. Lett. B **383**, 252 (1996).
- [6] E. Ramakrishnan et al., Phys. Rev. Lett. **76**, 2025 (1996).
- [7] T. Bauman et. al., Nucl. Phys. **A635**, 428 (1998).
- [8] D. Santonocito and Y. Blumenfeld, Eur. Phys. J. A **30**, 183 (2006).
- [9] A. Schiller and M. Thoennessen, At. Data Nuc. Data Tables **93**, 549 (2007).
- [10] G.F. Bertsch, P.F. Bortignon and R.A. Broglia, Rev. Mod. Phys. **55**, 287 (1983).
- [11] E. Liparini and S. Stringari, Phys. Rep. **175**, 103 (1989).
- [12] F.L. Braghin, D. Vautherin, Phys. Lett. **B333**, 289 (1994).
- [13] F.L. Braghin, D. Vautherin, A. Abada, Phys. Rev. C **52**, 2504 (1995).
- [14] S. Ayik, K. Bozkurt, A. Gokalp and O. Yilmaz, Acta Phys. Polonica, B **39**, 1413 (2008).
- [15] V.M. Kolomietz, A.B. Larionov and M. Di Toro, Nucl. Phys. **A613**, 1 (1997).
- [16] M. Di Toro, V.M.Kolomietz and A.B. Larionov, Phys. Rev. C **59**, 3099 (1999).
- [17] A.B. Larionov, M. Cabibbo, V. Baran, M. Di Toro, Nucl. Phys. **A 648**, 157 (1999).
- [18] V. Baran, M. Colonna, M. Di Toro and A.B. Larionov, Nucl. Phys. **A 649**, 185c (1999).
- [19] A.A. Abrikosov and I.M. Khalatnikov, Rept. Prog. Phys. **22**, 329 (1959).
- [20] G. Baym and C. Pethick, *Landau Fermi-liquid theory* (New York, John Wiley and Sons, 1991).
- [21] J. Wambach, Rep. Prog. Phys. **51**, 989 (1988).
- [22] S. Kamerdzhiev, J. Speth and G. Tertychny, Nucl. Phys. **A624**, 328 (1997).
- [23] M. Bender, P.-H. Heenen and P.-G. Reinhard, Rev. Mod. Phys. **75**, 121 (2003).
- [24] S. Stringari, Ann. of Phys. **151**, 35 (1983).

- [25] W.E. Ormand, P.F. Bortignon and R.A. Broglia, Phys. Rev. Lett. **77**, 607 (1996).
- [26] G. Gervais, M. Thoennessen and W. E. Ormand, Phys. Rev. **C58**, R1377 (1998).
- [27] L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Part 1 (Pergamon Press Ltd., Oxford, 1980).
- [28] E.M. Lifshits and L.P. Pitaevsky, *Physical kinetics*, (Pergamon Press, Oxford - New York - Seoul - Tokyo, 1993).
- [29] V.M. Kolomietz and H.H.K. Tang, Phys. Scripta **24**, 915 (1981).
- [30] K.-F. Liu, H. Luo, Z. Ma, O. Shen and S.A. Moszkowski, Nucl. Phys. **A 534**, 25 (1991).
- [31] G. A. Brooker and J. Sykes, Ann. Phys. **61**, 387 (1970).
- [32] S. Ayik, O. Yilmaz, A. Gokalp and P. Schuck, Phys. Rev. C **58**, 1594 (1998).
- [33] V.M. Kolomietz, V.A. Plujko and S. Shlomo, Phys. Rev. C **54**, 3014 (1996).
- [34] S. Ayik and D. Boilley, Phys. Lett. **B276**, 263 (1992); **B284**, 482E (1992).
- [35] G.Q. Li and R. Machleidt, Phys. Rev. C **48**, 1702 (1993); Phys. Rev. C **49**, 566 (1994).
- [36] V.M. Kolomietz and S. Shlomo, Phys. Rev. C **64**, 044304 (2001).
- [37] H. Krivine, J. Treiner and O. Bohigas, Nucl. Phys. **A 336**, 155 (1980).
- [38] O. Bohigas, A.M. Lane and J. Martorell, Phys. Rep. **51**, 267 (1979).
- [39] V.M. Kolomietz, S.V. Lukyanov and O.O. Khudenko, Ukr. Phys. Journ. **52**, 546 (2007).
- [40] A. Kolomiets, V.M. Kolomietz and S. Shlomo, Phys. Rev. C **59**, 3139 (1999).
- [41] V.M. Kolomietz, A.G. Magner and V.A. Plujko, Z. für Phys. A **345**, 131, 137 (1993).
- [42] V.M. Kolomietz and S. Shlomo, Phys. Rep. **390**, 133 (2004).
- [43] W.D. Myers and W.J. Swiatecki, Ann. Phys. **84**, 186 (1974).
- [44] A. Bohr and B. Mottelson, *Nuclear Structure* (W. A. Benjamin, New York, 1975), Vol. 2.
- [45] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, New-York, 1980).
- [46] B. L. Berman and S. C. Fultz, Rev. Mod. Phys. **47**, 713 (1975).
- [47] I. Hamamoto, H. Sagawa and X.Z. Zhang, Phys. Rev. C **56**, 3121 (1997).

- [48] W. D. Myers, W. J. Swiatecki, T. Kodama, L. J. El- Jaick and E. R. Hilf, Phys. Rev. **C15**, 2032 (1977).
- [49] J. Blocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk and W. J. Swiatecki, Ann. Phys. **113**, 330 (1978).
- [50] A. J. Sierk, S. E. Koonin and J. R. Nix, Phys. Rev. **C17**, 646 (1978).
- [51] J. Nix and A. J. Sierk, Phys. Rev. **C21**, 396 (1980).